CHAPTER 3
DERIVATIVES (continued)

3.3. RULES FOR DIFFERENTIATION

A. The derivative of a constant is zero: $\frac{d}{dx}[c] = 0$

B. The Power Rule: $\frac{d}{dx}[x^n] = n\, x^{(n-1)}$

C. The Constant Multiple Rule: $\frac{d}{dx}[c * f(x)] = c * f'(x)$

D. The Sum and Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

E. Note that we can now find the derivative of any polynomial easily!
F. The second derivative is just the derivative of the derivative. It has many notations.
G. Assignment: P. 120 {1 – 10}

H. The Product Rule: $\frac{d}{dx}[f(x) * g(x)] = f(x) * g'(x) + g(x) * f'(x)$

I. The quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) * f'(x) - f(x) * g'(x)}{g^2(x)}$

ho d[hi] minus hi d[ho] over ho ho
J. Notes on the quotient Rule:
1. Sometimes a quotient is really just a scalar multiple.
2. Sometimes a quotient can be written as a product.

K. Higher order derivatives and their notations.
L. Higher order derivatives on the calculator.
M. Applications are easier with theorems:
1. Making piece-wise defined functions continuous and differentiable.
2. Tangent and normal lines revisited.

N. Assignment: p. 120 {11, 12, 13, 14, 16, 17, 23, 24, 25, 27, and 30}

3.4. VELOCITY AND OTHER RATES OF CHANGE

A. In general, the instantaneous rate of change of a function at a point *a* is $f'(a)$.
B. Definitions:
1. *Displacement* is the change in position.
2. Motion along a line is called *rectilinear motion*.

C. Since the average velocity is the (change in distance)/(change in time), it is also the (displacement)/(change in time).

D. Concerning motion – and in particular, rectilinear motion - the (instantaneous) velocity is the derivative of the position function with respect to time. The sign of the velocity indicates direction of motion.

E. Definitions:

1. *Speed* is the absolute value of velocity.
2. *Acceleration* is the derivative of the velocity function w.r.t. time. It measures the rate of change of the velocity w.r.t. time and is the second derivative of the position function.

F. A comparison of signs on position, velocity, and acceleration for rectilinear motion:

	Negative	*Zero*	*Positive*
Position	Left / below the origin	ground zero / the origin	right / above the origin
velocity	moving left / down	not moving / stopped	moving right / up
acceleration	slowing down if v(t) > 0 speeding up if v(t) < 0	constant motion	speeding up if v(t) > 0 slowing down if v(t) < 0

G. Note that an object is speeding up if v(t) and a(t) have the same sign. It is slowing down if they have the opposite signs. We will include the end points for "speeding up" or "slowing down" because it is a comparison, but the A.P. folks don't care if you include them or not.

H. We will consider projectile motion now as a vertical form of rectilinear motion.

I. Assignment: P. 129 {2, 3, 5, and 6}

J. Formulas for projectile motion on the Earth that take gravity into account.

1. Definitions:
    a. h(t) is the height
    b. v_0 is the initial vertical velocity (at time zero)
    c. h_0 is the initial height. (at time zero)
2. Formulas:
    a. In terms of feet and seconds: $h(t) = -16t^2 + v_0t + h_0$
    b. In terms of meters and seconds: $h(t) = -4.9t^2 + v_0t + h_0$

K. The derivative is the rate of change function for all types of quantities.

L. Marginal means derivative in the world of economics. We must be careful with units.

M. Interpret rectilinear motion when given the graph of the velocity function.

N. It is possible to find where an object is speeding up or slowing down by looking at the graph of y = | v(t) |.

O. Assignment: P.130 {8, 9, 10, 12, 20, 23, and 25}

3.5. DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

A. Everything is based on two special limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad\qquad \lim_{x \to 0} \frac{1-\cos x}{x} = 0$$

B. From the above we can prove:

1. $\frac{d}{dx}[\sin x] = \cos x$

2. $\frac{d}{dx}$[cos x] = - sin x

C. The Quotient Rule allows us to prove the remaining trig derivatives.

1. $\frac{d}{dx}$[tan x] = $\sec^2$ x
2. $\frac{d}{dx}$[cot x] = - $\csc^2$x
3. $\frac{d}{dx}$[sec x] = sec x tan x
4. $\frac{d}{dx}$[csc x] = - csc x cot x

D. Doing everything we have done in the past with trigonometric functions.

1. Tangent and normal lines.
2. The product rule and the quotient rule.
3. Piece-wise differentiability and continuity.
4. Higher order derivatives with occasional patterns.

E. Assignment: P. 140 {2 - 12 even, 28, 29, and 30}

3.6. CHAIN RULE

A) Consider the derivatives of y = $\frac{1}{g(x)}$ and y = $[g(x)]^2$

B) The chain rule for the derivative of composite function y = f(g(x)) is $\frac{dy}{dx}$= f ' (g(x))*g ' (x)

C) In terms of another notation, if u = g(x), then y = f(u), the derivative w.r.t. x is: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

D) The part that gets chained usually exists within parenthesis. You may want to re-write using parenthesis. An example y = $\sqrt{x^2 - 5}$ can be re-written as y = $(x^2 - 5)^{(1/2)}$

E) For nested parenthesis you have a multiple chains. An example: f(x) = $\sin^4(x^2+1)$

F) Assignment: P. 146 {2 – 31 do 2 and skip 2}

G) An introduction to parametric curves and equations in all of their glory.

H) The chain rule allows us to establish the derivative of parametric equations: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

I) Finding the equation of the line tangent to the curve at a point defined by parametric equations.

J)The calculator can handle derivatives of parametric equations.

K) Working with a chart like problem #57 is another way to test your understanding of the chain rule.

L) Assignment: P. 146 {45 - 48, and 56}

3.7. IMPLICIT DIFFERENTIATION

A) An implicit function is one that is not solved for y in terms of x.
B) To perform implicit differentiation w.r.t. x, just take the derivatives of all variables using the theorems from the past, just stick a d(whatever)/dx on the end of the non x variables. The chain rule allows us to do this.
C) Now we can find the slopes of non-functions such as conic sections.
D) At times you must factor out and solve for $\frac{dy}{dx}$.
E) Finding horizontal and vertical tangents of implicit functions.
F) Look out for products and quotients, they are the downfall of many a student.
G) When taking the second derivative of an implicit function you will have to plug in the first derivative.
H) We can use implicit differentiation to prove the chain rule for rational exponents.
I) Unfortunately, the TI-89 does not do implicit differentiation. There exists a program/download for the calculator that enables it to do so.
J) Assignment P.155 {2 - 18 even, 19, 20, 23, 26, 27, 30, 41, and 42}

3.8. DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

A) Recall how to compute the trig(trig inverse).
B) We use implicit differentiation to prove the six inverse trig functions:

$$\frac{d}{dx}[\text{arcsin x}] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\text{arccos x}] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\text{arctan x}] = \frac{1}{x^2+1} \qquad \frac{d}{dx}[\text{arccot x}] = \frac{-1}{x^2+1}$$

$$\frac{d}{dx}[\text{arcsec x}] = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[\text{arccsc x}] = \frac{-1}{|x|\sqrt{x^2-1}}$$

C) Note that three are distinctly different while the other three are their opposites.
D) Note that y = arcsec x and y = arccsc x always have a positive slope. This is the reason for the absolute value. We can generate the absolute value of the derivative of arccsc(x) by considering the derivative of arcsin(1/x).
E) Applications involve rates of change of angles with respect to time. Units are in radians/time. These will be considered in the next chapter.
F) Note the interesting relationship between the slope of a function and the slope of its inverse: If (a, f(a)) is a point on y = f (x) at x = a, the derivative of the inverse of f at x = a is the reciprocal of the derivative of the function evaluated at x = f (a). See a picture and the formula on the bottom of page 57.
G) Assignment: P. 162 {2 - 19 do 2, skip 2; 20, 23}

3.9. DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

A) Use your calculator to guess what the derivative of $y = e^x$ is.

B) Most of the time we use it chained up: $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

C) For other bases we can convert to e, which gives us the formula: $\frac{d}{dx}[a^u] = a^u (\ln a) \frac{du}{dx}$

D) Recall the formulas for

1. Compound Interest: $A = P\left(1+\frac{r}{n}\right)^{nt}$
2. Continuous Compound Interest: $A = Pe^{rt}$
3. Exponential growth or decay: $A = Ae^{kt}$ or $A = A(b)^{t/c}$
4. Logistic growth: $P = \frac{M}{1+Ae^{-kt}}$

E) We can now investigate *how fast* your money is growing or your car is depreciating.

F) We can also consider the rate of growth or decay in nature. We will do all of this officially in chapter six.

G) With our friend implicit differentiation we can find the derivative of the inverse of e^x

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$$

H) Why stop with the natural logarithm when we can prove: $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u}\frac{du}{dx}$

I) We now have virtually all of the derivative formulas we will use in this course!

J) Playing log games (expanding as much as possible) will make taking the derivatives of logs much easier.

K) Logarithmic differentiation (not in the A.P. curriculum) introduces a log to make a differentiation possible.

L) Assignment: P. 170 {2 - 43 do 2 and skip 2, 44, and 47}

Mandatory Review: P. 172 {2 - 14 even, 20, 21, 27, 35, 38, 40, 47, 52, 67, 71, 73}
1997 BC Multiple Choice: {2, 4, 5, 6, 7, 10, 78, 79}